Abstracts for MAIA 2007

The limits of bivariate Lagrange projectors

Carl de Boor∗ and Boris Shekhtman
Eastsound, Washington

In a talk in Norway in 2003, the first author conjectured that any finite-rank ideal projector (i.e., finite-rank linear projector on the space of polynomials in d variables whose kernel is a polynomial ideal, i.e., a linear space also closed under pointwise multiplication by polynomials) is the (pointwise) limit of Lagrange projectors (i.e., projectors whose kernel consists of all polynomials that vanish on a certain finite set).

The chair of that talk's session, Geir Ellingsrud, pointed out to that first author (afterwards :-) that this conjecture must be wrong for $d > 2$ but may be correct for $d = 2$, and both these statements were subsequently proved by the second author, using tools from algebraic geometry.

The two authors are therefore pleased to present here a proof of the conjecture for $d = 2$ that uses nothing more than linear algebra.

Transfinite mean value interpolation in 3D

Solveig Bruvoll∗ and Michael Floater
Oslo, Norway

In this talk we study mean value interpolation over volumetric domains of arbitrary topology. We derive conditions on the boundary of the domain to guarantee interpolation when the data is continuous. By deriving the normal derivative of the interpolant and of a mean value weight function we construct a transfinite Hermite interpolant.

GC configurations with high defect

J. M. Carnicer∗, C. Godés
Zaragoza, Spain

The Geometric Characterization (GC), introduced by Chung and Yao [5], characterizes the sets of nodes such that the Lagrange polynomials are products of factors of first degree. In order to classify GC sets the defect of a configuration was introduced in [1], based on the number of lines containing more nodes than the degree. From the definition the defect is a nonnegative integer not greater than $n + 2$. A conjecture by Gasca and Maeztu on GC sets implies that the defect is not greater than $n - 1$ (see [2]). A complete description has been given for sets of defect 0, 1 and 2 and a recent research [3] allows us to describe all GC sets with defect 3 Assuming a conjecture by Gasca and Maeztu on GC sets (see [2]), a full description of GC sets with defect $n - 1$, as generalized principal lattices has been given in [3]. We prove that, if the conjecture of Gasca and Maeztu holds, then it is impossible to find GC sets with defect greater than 3 and less than $n - 1$. This result completes the description of all possible GC sets.

References

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[2] J. M. Carnicer, M. Gasca, On Chung and Yao's geometric characterization for bivariate polynomial interpolation, in *Curve and Surface Design: Saint-Malo 2002*, Tom Lyche, Marie-Laurence Mazure and Larry L. Schumaker (eds.), Nashboro Press, Brentwood TN, 2003, 21–30.

[3] Carnicer, J. M. and Godés, C., Geometric Characterization and generalized principal lattices, *Journal of Approx. Theory*, **143** (2006), 2–14.

[4] J. M. Carnicer, C. Gods, Geometric characterization of Configurations with Defect Three, in *Curve and Surface Fitting: Avignon 2006*, Albert Cohen, Jean-Louis Merrien and Larry L. Schumaker (eds.), Nashboro Press, Brentwood TN, 2007, 61–70.

[5] Chung, K. C. and Yao, T. H., On lattices admitting unique Lagrange interpolations, *SIAM J. Numer. Anal.* **14** (1977), 735–743.

Full rank interpolatory subdivision schemes with positive definite symbols

Costanza Conti∗, Mariantonia Cotronei, and Tomas Sauer
Florence, Italy

In this talk we consider full rank interpolatory vector subdivision schemes with positive definite matrix symbols and show how to obtain an associated cardinal refinable function and an associated iterative scheme. However, both the associated refinement equation and the associated "subdivision" scheme are different from the classical concepts.

Hermite subdivision schemes: a geometric construction

Paolo Costantini∗ and Carla Manni
Siena, Italy

In this talk we describe a family of non-stationary, non-uniform schemes for 2-points Hermite subdivision. Two are the main ingredients of this new approach: some recent results on the Bernstein-Bézier representation for weak Tchebycheff spaces and a geometric construction of the subdivision steps. The main advantages consist in extra smoothness conditions, which in turn produce highly regular limit curves, and in an elegant structure of the subdivision –described by three de Casteljau type matrices. As a by-product, the scheme is inherently shape preserving and allows a tensor-product extension.

A tension approach to controlling the shape of cubic spline surfaces

Oleg Davydov∗ and Carla Manni
Strathclyde, UK

We propose a tensioned (parametric) version of the FVS finite element to control the shape of the composite surface and remove artificial oscillations, bumps and other undesired behaviour. In particular, this approach is applied to C1 cubic spline surfaces over a four-directional mesh produced by two-stage scattered data fitting.

Transfinite mean value interpolation

Christopher Dyken∗ and Michael Floater
Oslo, Norway

Transfinite mean value interpolation has recently emerged as a simple and robust way to interpolate a function f defined on the boundary of a planar domain. In this talk we study basic properties of the interpolant, including sufficient conditions on the boundary of the domain to guarantee interpolation when f is continuous. Then, by deriving the normal derivative of the interpolant and of a mean value weight function, we construct a transfinite Hermite interpolant, and discuss various applications.

Approximation of univariate set-valued functions

Nira Dyn
Tel-Aviv, Israel

Constructing tight frames of multivariate functions

Say Song Goh, Tim N. T. Goodman∗ and S. L. Lee
Dundee, UK

Tight frames offer many of the advantages of orthonormal systems while allowing more flexibility. For functions satisfying general non-uniform, non-stationary refinement equations, we give a method of constructing associated tight frames. One example will be given for linear functions on arbitrary triangulations with edge mid-point subdivision, which can be applied for surfaces of arbitrary topology. Another example concerns Powell-Sabin elements on a six-direction mesh and preserves symmetry by using the theory of circulant matrices.

Generalized Multivariate Refinable Hermite Interpolants

Bin Han
Edmonton, Alberta

Multivariate refinable Hermite interpolants are of interest in approximation theory, CAGD and wavelet analysis. In this talk, we shall introduce a notion of generalized multivariate refinable Hermite interpolants with additional flexibility, which include refinable Hermite interpolants as a special case and which are of interest in sampling theory, CAGD and numerical algorithms. We obtain a complete characterization for generalized refinable Hermite interpolants with a general isotropic dilation matrix. Systematic construction is given for univariate generalized refinable Hermite interpolants. Some bivariate examples are also presented to illustrate the theory. The notion of generalized refinable Hermite interpolants also allows us to recover a known interesting example of bivariate spline (piecewise linear) refinable function vectors. Some connections of multivariate splines to multivariate refinable function vectors will be discussed.

Zonotopal algebra

Olga Holtz
Berkeley, California

A wealth of geometric and combinatorial properties of a given linear map X of $\mathbb{R}^N$ is captured in the study of its associated zonotope $Z(X)$, and, by duality, its associated hyperplane arrangement $\mathcal{H}(X)$. We associate X with three algebraic structures, referred herein as *external, central, and internal.* Each algebraic structure is given in terms of a pair of homogeneous polynomial ideals in n variables that are dual to each other: one encodes properties of the arrangement $\mathcal{H}(X)$, while the other encodes by duality properties of the zonotope $Z(X)$. The central pair is known from the works of Dahmen, Micchelli and others in 1980s and 1990s, while the other two are new. All three algebraic structures are defined purely in terms of the combinatorial structure of X, but are subsequently proved to be equally obtainable by applying suitable algebro-analytic operations to either of $Z(X)$ or $\mathcal{H}(X)$. The theory is universal in that it requires no assumptions on the map X, and has numerous applications in algebra, combinatorics, and approximation theory.

Lattices on simplicial partitions

Gasper Jaklic*, Jernej Kozak, Marjeta Krajnc,
Vito Vitrih, Emil Zagar
Ljubljana, Slovenia

In contrast to the univariate case, uniqueness of the solution of a multivariate Lagrange polynomial interpolation problem depends not only on the fact that interpolation points should be distinct but also on their geometry. Lattices are perhaps the most often used configurations of prescribed interpolation points on simplices in the domain.

In this talk, $d+1$-pencil lattices on simplicial complexes in $\mathbb{R}^d$ will be considered. The explicit representation of a lattice on a simplex, based upon barycentric coordinates, will be presented. This enables us to construct lattice points in a simple way and carries over to simplicial partitions in a natural way.

Multivariate Bernstein Basis Polynomials and Their Kernels

Kurt Jetter
Hohenheim, Germany

We study properties of the sequence of kernel functions

$$T_n(\mathbf{x},\mathbf{y}) = \sum_{|\alpha|=n} B_\alpha(\mathbf{x})\ B_\alpha(\mathbf{y})\ , \quad n = 0, 1, 2, \ldots$$

and applications thereof. Here, $B_\alpha(\mathbf{x}) = \binom{n}{\alpha}\mathbf{x}^\alpha$, for $|\alpha| = n$, denotes the Bernstein basis polynomials of degree n on the standard d-dimensional simplex $\mathbf{S}^d$. Thus, $\mathbf{x} = (x_0, x_1, \ldots, x_d)$, with $x_0 = 1 - x_1 - \cdots - x_d$, and $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_d) \in \mathbb{Z}_+^{d+1}$ with $|\alpha| = \alpha_0 + \alpha_1 + \cdots + \alpha_d$. Except for a factor $(n+d)!/n!$, these are the kernel functions of the d-variate Bernstein-Durrmeyer operator of degree n.

In particular, we will address the property of the sequence $\big(T_n(\mathbf{x},\mathbf{y})\big)_{n\geq 0}$ being completely monotonic, for any fixed $\mathbf{x}, \mathbf{y} \in \mathbf{S}^d$.

The talk is based on joint work with Elena Berdysheva and Joachim Stöckler.

References:

[1] E. Berdysheva, K. Jetter, and J. Stöckler, Durrmeyer operators and their natural quasi-interpolants, in: *Topics in Multivariate Approximation and Interpolation* (K. Jetter et al., eds.), pp. 1–21, Elsevier, Amsterdam, 2006.

[2] E. Berdysheva, K. Jetter, and J. Stöckler, New polynomial preserving operators on the simplex: direct results, *J. Approx. Theory* **131** (2004), 59–73.

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On multivariate Bernstein polynomials and optimal positive linear operators on an m-simplex

Francesco-Javier Munoz-Delgado
Jaen, Spain

We consider positive linear operators, L, that uses values $\{f\left(\frac{i_1}{m}, \ldots, \frac{i_n}{m}\right)\}$ with $i_1, ..., i_n \in \{0, 1, ..., m\}$ and $\sum_{h=1}^{n} i_h \leq m$. We prove that, if L fixes linear polynomials and there exists p_2 a eigenfunctions, polynomial of degree 2, $Lp_2 = \lambda_2 p_2$, then

$$\lambda_2 \leq \frac{m-1}{m} = \lambda_2(B_{m,n})$$

where $B_{m,n}$ is the classical m-th Bernstein operator on a n-dimensional simplex.

On multivariate Chebyshev polynomials and their applications

Hans Munthe-Kaas
Bergen, Norway

We consider families of multivariate Chebyshev polynomials as originally defined by Koornwinder ('74) and later generalized by Eier & Lidl ('75–'82) and Hoffman & Withers ('88). These multivariate polynomials share most of the beautiful properties of the classical univariate case, such as near optimal Lebesgue constants for the interpolation problem and existence of fast FFT based algorithms for interpolation, derivation and integration. Our interest in these polynomials arises from the desire to develop fast and accurate approximation algorithms for triangular domains.

On representations of rational surfaces

Juan-Manuel Peña
Zaragoza, Spain

Some recent advances on shape preserving and stability properties of rational surfaces are presented. Computational aspects of the corresponding evaluation algorithms are analyzed.

Approximate ideals and approximate varieties

Tomas Sauer
Gießen, Germany

It is well–known that for any finite set of points in $\mathbb{R}^d$ there exists a nontrivial polynomial $f \in \mathbb{R}[x_1, \dots, x_d]$ such that f vanishes at these points. However, from a computational point of view such implicit interpolants are not very useful as they are very sensitive to even small perturbations of the points. A more stable approach is to use polynomials from an *approximate ideal* which do not vanish at the points but only assume very small absolute values there.

Such a concept nicely connects to H–bases as polynomials of small degree are usually the numerically more stable and more easily managable ones.

Jackson- and Bernstein-type inequalities for robust norm equivalences

Karl Scherer
Bonn, Germany

In additive multilevel-methods *norm equivalences* with respect to bilinear forms associated with elliptic operators play a decisive role. These are established via Jackson- and Bernstein-type inequalities in weighted norms. We address this question for some examples where emphasis is laid on the constants in the equivalences. They are crucial for the stability of iteration methods for the stiffness matrix in the corresponding Ritz-Galerkin equations, thus for the robustness of the numerical method.

Curve intersection and Bézier clipping

Christian Schulz
Oslo, Norway

In the first part of this talk we will consider three different methods for computing intersections between parametric spline curves: Newton's method, a recently developed method based on intersections of control polygons and Bézier clipping. All three methods have in common that if they find an intersection, they will find it with a quadratic convergence rate for transversal intersections. For the first two methods formal proofs of the quadratic convergence rate are known, whereas for Bézier clipping the rate was based on numerical observations. In the second part of the talk we will show how to prove the quadratic convergence rate and what consequences we can draw from that.

Using TV-Stokes equations for digital image denoising and restoration

Xue-Cheng Tai
Bergen, Norway

In this talk, we shall try to summarize the work we have done recently about using partial differential equations for image analysis and processing, we shall concentrate on noise removal and restoration. We shall start by the well-known total variation denoising techniques. From numerical experiments and some analysis, we show that better properties can be kept if we use some modified higher oder partial differential equations. In the end, we show that TV-Stokes equations is a naturally choice for noise removal and image inpainting. However, the equations we derive here is not coming from fluid mechanics, but from some geometrical considerations for digital images. This talk is based on joint works with: S. Osher, R. Holm and T. Rahman. See `www.mi.uib.no/~tai`.

Nonlinear interpolatory subdivision

Johannes Wallner
Graz, Austria

In recent years the 'method of proximity' which means considering nonlinear subdivision rules as perturbations of linear ones, has been successfully applied to interpolatory schemes. G. Xie, T. Yu, and P. Grohs have obtained general smoothness equivalence results in the univariate and regular multivariate cases. The method of proximity works well for smoothness, but also for showing facts concerning coefficient decay in 'lazy' wavelet transforms based on interpolatory subdivision schemes. Joint work with P. Grohs.

On the deviation of a parametric polynomial interpolant from its data polygon

Emil Zagar
Ljubljana, Slovenia

When fitting a parametric polynomial curve through a sequence of points, it is important in applications that the curve should not exhibit unwanted oscillations. In this talk we study the local and global deviations of the polynomial interpolant from the data polygon relative to the lengths of the polygon edges, focusing on the simple parameterization in which each parameter interval length is some power between 0 and 1 of the length of the chord between the two corresponding data points.

Piecewise polynomial differential forms

Ragnar Winther
Oslo, Norway

A Solution to Donoho's conjecture on Subdivision Scheme of Manifold Valued Data

Thomas Yu
Philadelphia, Pennsylvania